ONE APPROACH TO CONSTRUCTING A METHOD FOR DESIGNING MODEL HEAT SHIELDS

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This paper proposes an engineering approach to construction of a method for designing multilayer heat shields that is based on iterative use of rigorous and simplified mathematical models and permits effective assembly synthesis on the basis of the condition that specified constraints on temperature conditions be satisfied.

The optimum-mass solution to the problem of designing a one-dimensional assemblage (stack) with specified structure that will be exposed to a high-temperature environment and is characterized by constraints on the temperature conditions in individual zones is the solution of a problem of the type:

$$M = \sum_{j=1}^{n} \rho_{\text{var},j} h_{\text{var},j} \to \min_{\overline{h}_{\text{var}}}, \tag{1}$$

$$T_{\text{con},i} \leqslant \hat{T}_{\text{con},i}, \quad i = \overline{1, m}, \tag{2}$$

$$h_{\text{var},j} \geqslant \check{h}_j, \quad j = \overline{1, n}. \tag{3}$$

Here the temperature conditions are described within the framework of the one-dimensional Fourier equation [1].

The penalty function method [2] is used to reduce problem (1)-(3) to the unconditional minimization problem

$$F = \sum_{j=1}^{n} \rho_{\text{var},j} h_{\text{var},j} + \sum_{i=1}^{m} a_i \max(0, T_{\text{con},i} - \hat{T}_{\text{con},i}) + \sum_{j=1}^{h} b_j \max(0, \check{h}_j - h_{\text{var},j}) \to \min_{\overline{h}_{\text{var}}}. \tag{4}$$

Various aspects of the development of methods for solution of this sort of problem were considered in [3, 4]. However, the above techniques have not found wide application in investigation of practical shield design problems. Considerably more frequent use is made of a simplified approach to assemblage synthesis based on seeking that combination of thicknesses for the individual layers (whose total number is m) which assures satisfaction of conditions having the form

$$\varphi_i(h_{\text{var},1}, \ldots, h_{\text{var},m}) = T_{\text{con},i}(h_{\text{var},1}, \ldots, h_{\text{var},m}) - T_{\text{con},i} = 0, \quad i = \overline{1, m}. \tag{5}$$

The present study is devoted to methodological problems pertaining to construction of solutions for problems of the type formulated above.

One possible iterative approach to solution of problem (5) (which we will call algorithm 1) consists in performing a sequence of operations in each k-th iteration:

formation of an initial approximation $h_{\text{var},j}^{(k)}$ $(j=\overline{1,m})$ for the layer thicknesses sought;

calculation of the functionals $\varphi_i^{(k)}$ and their partial derivatives $\varphi_{h,i}^{(k)}$ with respect to the arguments $h_{\text{var},j}$ $(i, j=\overline{1,m})$ using the heat conduction equation;

determination of the layer thickness increments $\Delta h_{\text{var},j}^{(k)}$ satisfying the system of linear algebraic equations

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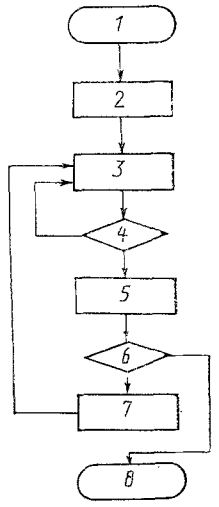

Fig. 1. Structure of search for problem solution. 1) Input of initial data; 2) initial formation of corrective links; 3) simplified mathematical model; 4) module for making decision as to whether to continue search for solution in inner loop; 5) rigorous mathematical model; 6) module for making decision as to whether to continue search for problem solution in outer loop; 7) current formation of corrective links; 8) exit from iterative process.

$$\sum_{j=1}^{m} \varphi_{h,i,j}^{(k)} \Delta h_{\text{var},j}^{(k)} = -\varphi_i^{(k)}, \quad i = \overline{1, m}, \tag{6}$$

obtained by linearization of system of equations (5).

The transition from the k-th to the k + 1-th iteration is made with equations having the form

$$h_{\text{var},j}^{(k+1)} = h_{\text{var},j}^{(k)} + \Delta_{h,j}^{(k)}, \quad j = \overline{1, m}, \quad k = 1, 2, \ldots; \tag{7}$$

$$\Delta_{h,j}^{(k)} = \begin{cases} \Delta h_{\text{var},j}^{(k)}, & |\Delta h_{\text{var},j}^{(k)}| \leqslant |\Delta \hat{h}_{\text{var},j}^{(k)}|, \\ \Delta \hat{h}_{\text{var},j}^{(k)}, & |\Delta h_{\text{var},j}^{(k)}| > |\Delta \hat{h}_{\text{var},j}^{(k)}|, \end{cases} \tag{8}$$

$$j = \overline{1, m};$$

$$\Delta \hat{h}_{\text{var},j}^{(k)} = \beta_h h_{\text{var},j}^{(k)} \operatorname{sign} (\Delta h_{\text{var},j}^{(k)}), \quad j = \overline{1, m}. \tag{9}$$

The iteration process is terminated at that iteration ℓ in which the condition

$$|\varphi_i^{(l)}| \leqslant \varepsilon_i, \quad i = \overline{1, m}$$

is satisfied.

Specific calculations made with this algorithm will be given below. the effectiveness of algorithm 1 is to a considerable extent determined by the choice of initial approximation. Moreover, special emphasis must be placed on the fact that use of adequately fitting mathematical models (particularly those that take into account the dependence of thermophysical properties on temperature and pressure, the nonlinearity of the boundary conditions, and the multidimensional nature of the problem) in this scheme often leads to expenditure of large amounts of computer time in order to obtain the solution sought.

When this sort of situation arises, one generally proceeds by utilizing simplified mathematical models of the process under investigation. However, alternatives must be sought if this approach does not yield a satisfactory solution to the problem (because of violation of restrictions on the permissible error in the solution found).

One effective way to overcome these difficulties was reported in [5], which describes investigation of mathematical models of radiative–convective heat transfer in high-temperature shock layers.

We can, utilizing this latter study, propose the following search structure (depicted in Fig. 1) for seeking a solution described by complex reentrant mathematical models (in [5], this reenterability was a product of the use of iterative methods for iteration processes with relatively slow convergence). The overall procedure for seeking a solution is

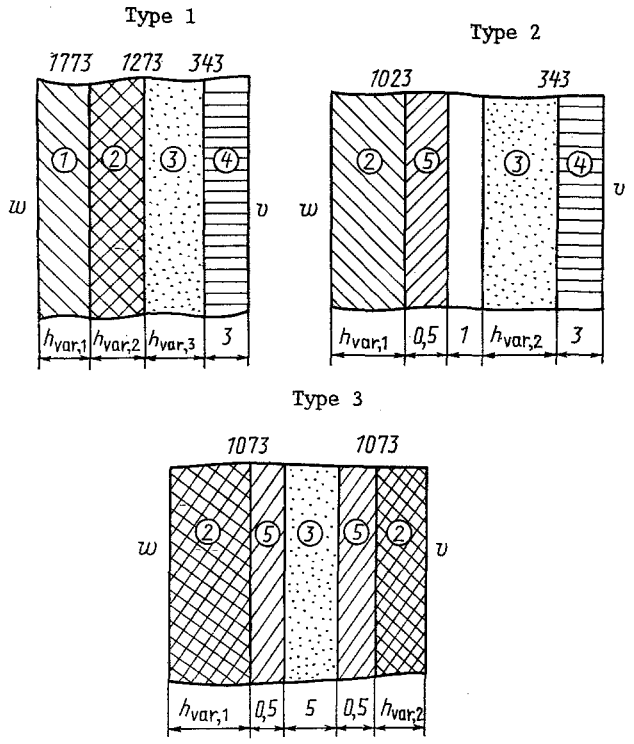

Fig. 2. Types of structural stacks. 1-5) Material numbers (see Table 1).

subdivided into two loops: linear and outer. The outer loop employs a materially simplified analog of the original mathematical model, and the number of solutions can be quite large. Each time that the solution search process exits into the outer loop, the accuracy of the solution obtained in the inner loop is analyzed (using a strict mathematical model of the phenomenon under investigation) and a certain number of corrective links is formed to eliminate the errors in the particular solution search phase that are associated with the simplified mathematical model in the inner loop. As a result, the problem solution can be taken as exact when the interations in the outer loop converge.

Use of this approach is effective only when the solution sought is obtained after a relatively small number of outer-loop iterations, which can in turn be assured only with a good choice of the simplified mathematical model for the process under investigation and of the principle utilized to construct the corrective links.

Construction of the numerical solution of the direct heat conduction problem in all stages of the present investigation is based on use of an implicit scheme for approximating the Fourier equation, with nonuniform partitioning of the domain of definition over the space coordinate and application of the trial-and-error method [6].

Simplification of the computation procedure in the inner loop of the dual-loop algorithm is based on use of thermophysical properties averaged over the temperature variation range under consideration ($\bar{c}$ and $\bar{\lambda}$) for each layer of the stack and on significant reduction of the number of integration steps over both the space and time coordinates.

The maximum admissible contact temperatures serve as the corrective links between the simplified and rigorous mathematical models.

Seeking the solution of the problem within the framework of this approach (which we will call algorithm 2) consists in performing the following sequence of operations in each p-th outer iteration:

formation of the initial approximations $\hat{T}^{(p)}_{con,\,i}$ $(i=\overline{1,m})$ for the maximum admissible temperatures in the controlled contacts used in the inner loop;

determination of the variable layer thicknesses $h^{(\ell),(p)}_{var,j}$ and the corresponding partial derivatives $\varphi^{(l),(p)}_{h,i,j}$ $(i, j=\overline{1,m})$, in complete conformity with algorithm 1;

TABLE 1. Properties of Materials

Material number	Relationship		Av. values		Density
	$\lambda(T)$	$c(T)$	$\overline{\lambda}$	$\overline{c}$	ρ
1	$30-5\overline{T}$	$1500+0,2T$	21	1900	2100
2	$(1+\overline{T}+\overline{T}^2)0,05$	$800+0,2T$	0,1	1000	200
3	$0,04+0,01\overline{T}+0,03\overline{T}^2$	$1800-T+400\overline{T}^2$	0,06	1300	100
4	$0,4+0,1\overline{T}$	$-400+5T$	0,45	1400	1800
5	$20+25\overline{T}$	$150+0,5T$	30	450	7880

TABLE 2. Formulas for Calculating Heat Transfer at Stack Surface w

q_w	$(\alpha/c_p)_w$	$I_{e,w}$	I_w
$(\alpha/c_p)_w(I_{e,w}-I_w)-$ $-\varepsilon_w\sigma T_w^4$	$0,03\exp\left[\left(3\frac{\tau-1000}{1000}-2\right)\times\right.$ $\left.\times\left(\frac{\tau-500}{500}\right)^2\right]$	$3\cdot10^5(1+100\times$ $\times\cos(\pi\tau/200))$	$954T_w+$ $+0,0862T_w^2$

TABLE 3. Formulas for Calculating Heat Transfer at Stack Surface v

Stack type	q_v	$(\alpha/c_p)_v$	$I_{e,v}$	I_v
1	$15(323-T_v)+\varepsilon_v\sigma\times$	—	—	—
2	$\times(333^4-T_v^4)$	—	—	—
3	$(\alpha/c_p)_v(I_{e,v}-I_v)-$ $-\varepsilon_v\sigma T_v^4$	$0,2(\alpha/c_p)_w$	$I_{e,w}$	$954T_v+0,0862T_v^2$

determination of the values of the functionals ψ_j:

$$\psi_j^{(p)}=\Theta_{\text{con},j}^{(p)}-\hat{\Theta}_{\text{con},j},\ j=\overline{1,\ m}, \tag{10}$$

and their partial derivatives $\psi_{h,i,j}^{(p)}$ from the arguments $h_{\text{var},j}$ $(i,j=\overline{1,m})$ in the rigorous formulation, with variable layer thicknesses $h_{\text{var},j}^{(\ell),(p)}$;

determination of the increments $\Delta h_{\text{var},j}^{(p)}$ in the layer thicknesses, which satisfy the system of linear algebraic equations

$$\sum_{j=1}^{m}\psi_{h,i,j}^{(p)}\Delta h_{\text{var},j}^{(p)}=-\psi_i^{(p)},\ i=\overline{1,\ m}, \tag{11}$$

obtained by linearization of system of equations (10);

determination of the increments for the maximum admissible temperatures in the controlled contacts utilized in the inner loop, from the equation

TABLE 4. Results Yielded by Solution of Stack Design Problem

Stack type	Initial approx. $\bar{h}^0_{var} \cdot 10^3$	Solution of problem $\bar{h}^*_{var} \cdot 10^3$	Computation time τ_c with algorithm	
			1	2
1	(5; 5; 10)	(17,7; 20,1; 35,2)	290	116
1	(20; 30; 30)	(17,7; 20,1; 35,2)	82	107
2	(5; 10)	(25,3; 32,4)	399	173
3	(10; 5)	(39; 5,9)	649	109

TABLE 5. Evaluation of Influence of Choice of Thermophysical Characteristics for Simplified Mathematical Model on Convergence Rate of Dual-Loop Algorithm

Elements of simplified mathematical models		Initial approx. $\bar{h}^0_{var}$	Solution of problem $\bar{h}^*_{var}$	Computation time τ_c
$\bar{\lambda}_2$	$\bar{c}_2$			
0,2	1000	(5; 5; 10)	(17,7; 20,1; 35,2)	115
0,1	1200	(5; 5; 10)	(17,7; 20,1; 35,2)	138

$$\Delta T^{(p)}_{con,i} = \sum_{j=1}^{m} \varphi^{(l);(p)}_{h,i,j} \Delta h^{(p)}_{var,j}, \quad i = \overline{1, m}, \tag{12}$$

which follows from (6).

The transition from the p-th to the p + 1-th outer iteration is made with formulas of the type

$$T^{(p+1)}_{con,i} = T^{(p)}_{con,i} + \Delta^{(p)}_{T,i}, \quad i = \overline{1, m}, \quad p = 1, 2, \ldots; \tag{13}$$

$$\Delta^{(p)}_{T,i} = \begin{cases} \Delta T^{(p)}_{con,i}, & |\Delta T^{(p)}_{con,i}| \leqslant |\Delta \hat{T}^{(p)}_{con,i}|, \\ \Delta \hat{T}^{(p)}_{con,i}, & |\Delta T^{(p)}_{con,i}| > |\Delta \hat{T}^{(p)}_{con,i}|, \end{cases} \tag{14}$$

$$i = \overline{1, m};$$

$$\Delta \hat{T}^{(p)}_{con,i} = \beta_T |T^{(p)}_{con,i} - T_0| \operatorname{sign} (\Delta T^{(p)}_{con,i}), \quad i = \overline{1, m}. \tag{15}$$

The iteration process terminates at the r-th iteration step, where the conditions

$$|\psi^{(r)}_i| \leqslant \delta_i, \quad i = \overline{1, m}$$

are satisfied.

The partial derivatives $\varphi_{h,i,j}$ and $\psi_{h,i,j}$ $(i, j = \overline{1,m})$ are computed by a numerical differentiation procedure.

Since theoretical investigation of the proposed dual-loop iteration algorithm is a complicated separate problem in the general case. We undertook to confirm its workability and effectiveness in an extensive numerical experiment.

We investigated various stack structures, considered both unilateral and bilateral heating with essentially nonlinear boundary conditions, and studied the influence exerted by the initial approximation and the elements of the simplified mathematical model (the constant thermophysical characteristics) on the convergence of the dual-loop algorithm.

As an illustration of these investigations, we will examine the results yielded by solution of the design problem for three typical shields (Fig. 2).

The admissible temperatures are specified near the corresponding controlled contacts. The thicknesses in mm are indicated for the invariant layers.

The thermophysical characteristics of the materials and the formulas for the heat transfer parameters used in the calculations are given in Tables 1-3. The emissivities of all the structural stacks taking part in radiative heat transfer were assumed to equal 0.8, while values of 0.2 and 0.5 respectively were used for the parameters β_h and β_T in Eqs. (9) and (15), which limit the rate of change of the parameters $h_{var,j}$ and $T_{con,i}$ in the iteration processes.

The basic results of this investigation, which was conducted with an Él'brus 1-2K computer, are given in Table 4. It can be seen that algorithm 2 had a computation time advantage over algorithm 1 in all cases except when the computations were made with a fortunately chosen initial approximation, whereupon the calculation procedures in question were more or less "equivalent."

As an example, Table 5 gives the results of design calculations made for a stack of the first type, with the other parameters of the simplified mathematical model different from those adopted in Table 1. The computations showed that the choice of a quite broad range of elements for the simplified mathematical model had little influence on the convergence rate of the dual-loop algorithm.

The results of this computer experiment enable us to state that the iteration process we have constructed converges for a wide variety of practical problems, regardless of the choice of the initial approximation and the approximate mathematical model.

Analysis of our computations shows that the proposed approach to construction of a solution method for the problem of designing a multilayer heat shield based on the condition that specified constraints on temperature conditions be satisfied is effective and quite simple to implement.

It should be noted that the optimal solution to the mass minimization problem in the formulation of (4), which was obtained by a direct Huck–Jeeves search [7], was found to coincide with the solutions found by procedures 1 and 2 for all the cases considered. This is still another argument in favor of the wide use in engineering practice of the simplified formulation of (5).

NOTATION

φ, ψ) functionals, K; T, Θ) temperatures calculated from simplified and rigorous models respectively, K; $\hat{T}$, $\hat{\Theta}$) maximum admissible temperatures utilized in inner and outer loops, K; T_{con}, Θ_{con}) temperatures of controlled contacts for simplified and rigorous models, K; T_0) initial assemblage temperature, K; $\hat{T} = T/1000$; q_w, q_v) incoming heat flux densities at stack boundaries w and v, W/m^2; c) specific heat capacity, J/(kg·K); $\bar{c}$) average specific heat capacity, J/(kg·K); λ) coefficient of thermal conductivity, W/(m·K); $\bar{\lambda}$) average coefficient of thermal conductivity, W/(m·K); ρ) material density, kg/m^3; $\bar{h}_{var} = (h_{var,1}, \ldots, h_{var,m})$ collection of variable stack layers, m; $\check{h}_j$) minimum admissible thickness of variable layer, m; ε_w, ε_v) emissivities of stack boundary surfaces; $I_{e,w}$, $I_{e,v}$) enthalpies of gas stream reduction at stack boundary surfaces, J/kg; I_w, I_v) enthalpies of gas stream reduction at temperatures of walls w and v, J/kg; $(\alpha/c_p)_w$, $(\alpha/c_p)_v$) heat transfer coefficients at surfaces w and v, kg/(m^2·sec); σ) Stefan–Boltzmann constant; τ) time, sec; τ_c) computation time; $\varepsilon_i = \delta_i = 0.01$) required accuracies of iteration processes in inner and outer loops; a_i, b_j) penalty coefficients. Index on thermophysical properties denotes number of material used (see Table 1).

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